Adaptive α-β Tracker for TWS Radar System

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Abstract: An adaptive α-β tracker is proposed for tracking maneuvering targets with a track-while-scan radar system. The tracker gain is updated on-line corresponding to the adjusted process noise variance which is obtained via time averaging of the process over a sliding window. The adjusted process noise variance is used to compute the maneuverability index for the tracker gain based on the steady-state Kalman filter equation for each epoch. It is shown via simulation that the proposed approach provides robust and accurate position estimates during the target maneuver while the performance of the conventional α-β tracker is shown much degraded.

Keywords: TWS, adaptive α-β tracker, maneuvering target tracking, Kalman filter

1. INTRODUCTION

Although various target tracking algorithms have been developed over the past half century, the α-β tracker developed in the mid 1950's is still widely used in the surveillance radar systems which require ability to track a great number of targets at the same time. This is because the α-β tracker requires minimal computational load due to its simplicity.

The α-β tracker is generally used for tracking targets under steady-state, stationary conditions in which the tracking problem is characterized by (a) constant track rate, (b) constant radar measurement noise variance, and (c) constant target maneuverability [1].

The α-β tracker can track a non-maneuvering target accurately with a Track-While-Scan (TWS) system with a uniform data rate and stationary measurement noise. However, when the target maneuvers, the quality of the position and velocity estimates provided by the α-β tracker can be degraded significantly and the target may even be lost since the filter uses only a linear prediction [2]. In practice, it is difficult to know the statistical properties of the target in advance, and each target has different dynamic characteristics. Also, radar measurement error varies according to range and angle in the Cartesian coordinate system.

In this paper, an adaptive α-β tracker is proposed whose gain is calculated on-line corresponding to the adjusted process noise. In order to adjust the process noise continuously, the covariance of the innovation process is updated via time averaging of the process over a sliding window and the fudge factor of the process noise variance is calculated using a steady-state Kalman filter equation for the process covariance at each epoch. The target maneuvering index is obtained from these results and the new α-β gain is calculated thereof. The performance of the proposed adaptive α-β tracker is compared with conventional α-β tracker in terms of the Root Mean Square Error (RMSE) and Normalized Position Error (NPE), where NPE is the ratio of the root mean square of position estimate error to the root mean square of measurement error [3].

2. OPTIMAL α−β TRACKER

2.1 Target Model

Consider a one dimensional, two state discrete time target motion model:

$$\begin{bmatrix} x_{k+1} \\ \dot{x}_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} v_k \tag{1}$$

where x_k is the position state at time k, $\dot{x}_k$ is the velocity state at time k, T is the constant track update period, and v_k is the unknown target maneuver, which is modeled by a zero mean, white Gaussian noise process with variance σ_v^2.

The observed target position by radar is

$$z_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} + w_k \tag{2}$$

where w_k is the measurement noise, which may be modeled by a zero mean, white Gaussian noise with variance σ_w^2.

2.2 α-β Tracker

The α-β tracker is based on the steady state Kalman filter for the ideal, two state motion model:

Prediction: $$\begin{bmatrix} \hat{x}_k^- \\ \hat{\dot{x}}_k^- \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{k-1}^+ \\ \hat{\dot{x}}_{k-1}^+ \end{bmatrix} \tag{3}$$

Correction: $$\begin{bmatrix} \hat{x}_k^+ \\ \hat{\dot{x}}_k^+ \end{bmatrix} = \begin{bmatrix} \hat{x}_k^- \\ \hat{\dot{x}}_k^- \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix} \left[z_k - \hat{x}_k^- \right] \tag{4}$$

where $\hat{x}_k^-$ and $\hat{\dot{x}}_k^-$ are the predicted position and velocity estimates, $\hat{x}_k^+$ and $\hat{\dot{x}}_k^+$ are the corrected position and velocity estimates, z_k is the measurement, and α and β are the tracking filter gains.

2.3 Optimal α-β Gain Selection

A means for selecting α-β gain was introduced by Kalata [4] who defined a variable λ, known as the tracking index, which is represented as follows:

$$\lambda = \frac{\sigma_v}{\sigma_w} T^2 \tag{5}$$

which is a function of the assumed target maneuverability variance and the radar measurement noise variance.

The coefficients for an optimal α-β tracker are given in terms of the tracking index by [5]:

$$\alpha = -\frac{1}{8}(\lambda^2 + 8\lambda - (\lambda + 4)\sqrt{\lambda^2 + 8\lambda}) \tag{6}$$

$$\beta = \frac{1}{4}(\lambda^2 + 4\lambda - \lambda\sqrt{\lambda^2 + 8\lambda}) \tag{7}$$

The relationship between α and β is given by

$$\beta = 2(2 - \alpha) - 4\sqrt{1 - \alpha} \tag{8}$$

The constraints on the α, β coefficients for the stability of the tracking filter are given by [6]:

$$0 < \alpha < 1 \tag{9}$$

$$\beta < 4 - 2\alpha \tag{10}$$

The optimal gain and steady state error covariance matrix can be represented as follows:

$$K_o = \begin{bmatrix} \alpha \\ \beta / T \end{bmatrix} \tag{11}$$

$$P_o = \begin{bmatrix} \alpha & \beta/T \\ \beta/T & \dfrac{\beta(2\alpha - \beta)}{2(1-\alpha)T^2} \end{bmatrix} \sigma_w^2 \tag{12}$$

3. ADJUSTMENT OF PROCESS NOISE

For the adjustment of the process noise, the normalized innovation should be monitored continuously for the detection of the target maneuver. The uncertainty in the maneuver is represented by a fudge factor ϕ_k for the process noise, which may be incorporated in the equation for the covariance of the innovations [7] as

$$S_k = H(\Phi P_{k-1}^+ \Phi^T + \phi_k Q)H^T + R \tag{13}$$

where S_k is the covariance of the innovation process, H is the observation matrix, Φ is the state transition matrix, P_{k-1}^+ is the updated state covariance matrix, Q and R are the covariance matrices of the process noise and measurement error, respectively.

In steady state, Eq. (13) may be written as follows:

$$S_k = H(\Phi P_o \Phi^T + \phi_k Q)H^T + R \tag{14}$$

where P_o is the state covariance matrix in the steady state.

It follows from Eq. (14) that

$$\phi_k = (S_k - H\Phi P_o \Phi^T H^T - R)(HQH^T)^{-1} \tag{15}$$

The time-varying innovation process covariance S_k in Eq. (15) can be estimated by a time-averaging of the process over a sliding window as follows:

$$\hat{S}_k = \frac{1}{N}\sum_{i=k-N}^{k} \tilde{z}_i \tilde{z}_i^T \tag{16}$$

where $\tilde{z}_k$ is the measurement residual ($\tilde{z}_k = z_k - \hat{z}_k$) at t_k, z_k and $\hat{z}_k$ are the actual and predicted measurements at t_k, respectively, and N is the length of the sliding window.

The fudge factor ϕ_k represents the estimate of the maneuverability variance, which should be semi-positive. The fudge factor obtained by Eq. (15) should be subject to that condition. If ϕ_k, calculated by Eq. (15), is negative, a pre-defined maneuverability variance σ_D^2 is used in its place. This can be represented as follows:

$$\hat{\sigma}_v^2 = \begin{cases} \phi_k & for\ \phi_k \geq 0 \\ \sigma_D^2 & for\ \phi_k < 0 \end{cases} \tag{17}$$

4. COMPUTER SIMULATION

In the computer simulation, the target maneuver is modeled by constant accelerations over pre-defined periods. The scenario of the simulated target motion is described in Table 1.

Table 1 The scenario of the simulated target motion

Mode	Target Motion	Period(sec.)	Remark
1	Constant Velocity	1 – 25	-
2	Constant Acceleration	26 – 35	5g
3	Constant Velocity	36 – 65	-
4	Constant Acceleration	66 – 75	-5g
5	Constant Velocity	76 – 100	-

The target trajectory is generated on a single axis with one-second sampling interval. The initial speed of the target is 120 m/s. The target speed varies according to the target motion to the maximum speed of about 610 m/s.

The true trajectory and the radar measurements are depicted in Fig. 1. The solid line represents the true trajectory, and the dotted line represents the radar measurements. It is assumed that the radar measurement error is known to be zero mean, white Gaussian noise with the standard deviation of 150m.

The effective length N of the sliding window, used for averaging the innovation, is 12. The performance of the proposed α-β tracker is affected by the window length.

The true velocity of the target and the velocity estimates of the conventional and proposed α-β trackers are depicted in Fig. 2. The solid line represents the true velocity, and the dotted line represents its estimate by the conventional tracker, and the velocity estimate of the proposed tracker is marked by a line with dashes and dots. It is seen from Fig 2 that the proposed tracker follows the target motion properly during the maneuvering periods whereas the conventional one tracks the target sluggishly with significant tardiness.

The RMS position and velocity estimation errors are depicted in Figs. 3 and 4. In Fig 3, the solid line represents the RMS errors of the radar measurements, and the dotted and dash-dotted lines represent the RMS errors of the position estimates of the conventional α-β tracker and the proposed α-β tracker, respectively. In Fig. 4, the RMS errors of the velocity estimates of the conventional tracker and the proposed tracker are represented by the solid line and the dotted line, respectively. The NPEs of the two trackers are depicted in Fig. 5. The NPEs of the conventional tracker and the proposed tracker are plotted by the solid and dotted lines, respectively.

It is shown from these results that the proposed α-β tracker tracks the target accurately during the maneuvering periods while the conventional tracker exhibits much degraded performance. The results demonstrate the robust and adaptive characteristics of the proposed tracker.

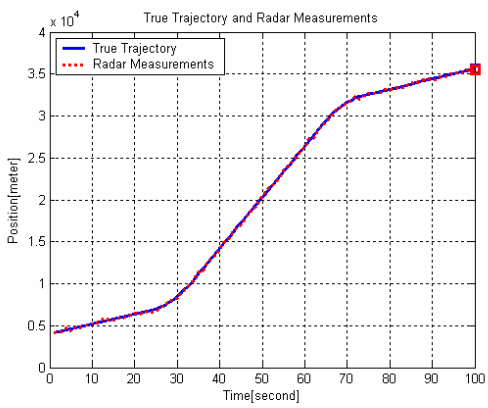

Fig. 1 Target Trajectory

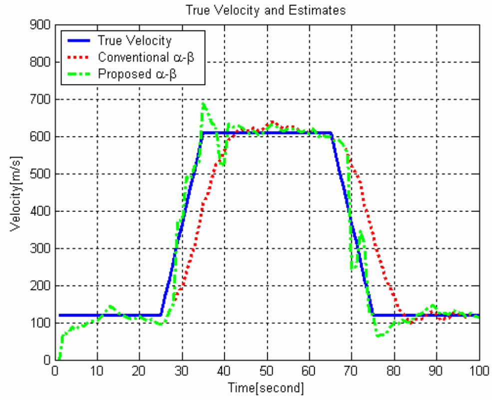

Fig. 2 True velocity and estimates

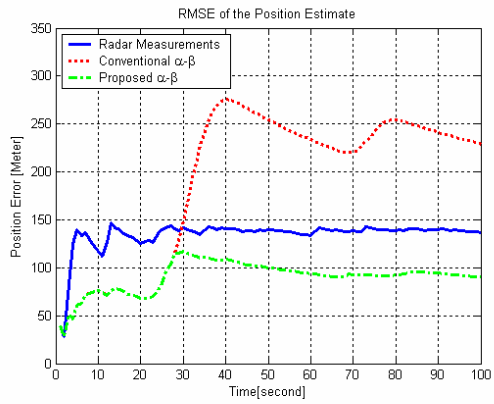

Fig. 3 RMSE of the position estimate

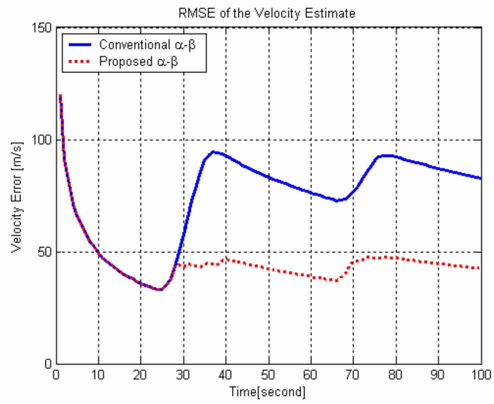

Fig. 4 RMSE of the velocity estimate

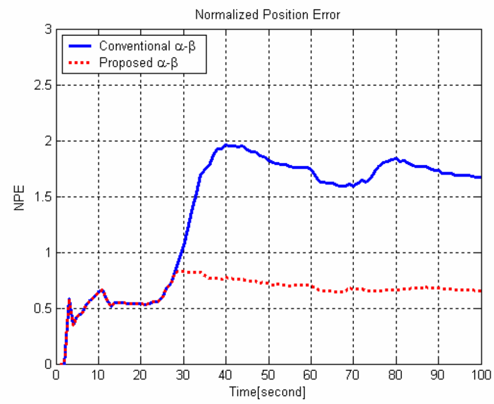

Fig. 5 NPEs

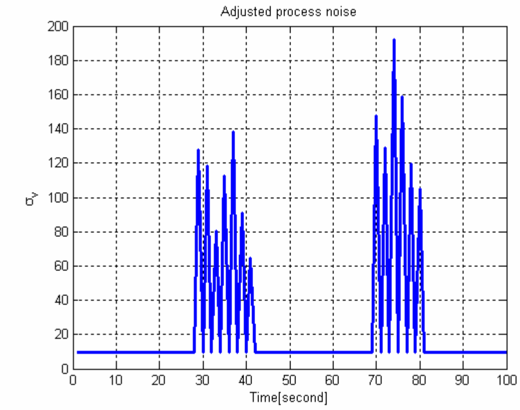

Fig. 6 Adjusted process noise.

The adjusted process noise which is calculated by the proposed α-β tracker is shown in Fig. 6. It is seen that the adjusted process noise follows the pattern of the target motion scenario. This is because the process noise is adjusted according to the fudge factor for the maneuver.

5. CONCLUSION

In this paper, an adaptive α-β tracker whose gain is calculated on-line based on the adjusted process noise was proposed for tracking maneuvering targets. It was shown via simulation that the proposed method follows the target successfully during maneuvers and provides more accurate and robust performance compared to the conventional α-β tracker.

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